Wave Analysis of Noise in Interconnected Multiport Networks

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Abstract —The noise performance of multiport networks of arbitrary topology is treated using wave analysis. This approach has advantages over other methods when using computer-aided design programs that are based on scattering parameters. In this paper we discuss the wave representation of noise in two-ports and passive multiports. We indicate how to compute the noise performance of an arbitrary network and we demonstrate the effectiveness of this approach with an example.

I. Introduction

As COMPUTER-AIDED DESIGN (CAD) increases for microwave circuit and system development, it is imperative that compatible analysis techniques be developed for the noise performance of these circuits and systems. Recently, Rizzoli and Lipparini [1] showed how the noise behavior of multiport networks of arbitrary topology may be analyzed in terms of impedance and admittance parameters. They extended the use of the usual four spot noise parameters defined by Haus [2], to handle circuits which include any kind of passive components introducing only thermal noise and any number of two-port devices.

In this paper, we approach this problem from the wave analysis point of view used by Rothe and Dahlke [3], Penfield [4], and Meys [5]. This approach is compatible with CAD software packages that are based on scattering parameters. One such package, called CAAMS, has been developed at the University of Massachusetts under sponsorship by Sanders Associates, Nashua, NH. SUPER-COMPACT, TOUCHSTONE, etc., are other commercially available packages that use scattering parameters in addition to the impedance and admittance parameters. The wave approach includes those advantages explained by Siegman [6] of using forward- and backward-traveling noise waves rather than equivalent voltage and current noise sources.

Analyzing the noise properties of microwave networks using the wave approach is certainly not original. Previous authors have modeled two-port networks where two correlated noise-wave sources are located at the input of the two-port. In analyzing noise in systems containing multiport networks, however, we choose to place one noise wave source at each network port. This approach allows us to conveniently compute the noise properties of the overall system in terms of the scattering parameters that are used to analyze the system's gain performance.

We discuss the wave representation of noise in a noisy two-port in Section II and the more general problem of noise in passive multiports in Section III. In Section IV, we show how the noise analyses can be applied to computing the noise performance of a microwave network of arbitrary topology. We conclude the paper with an example in Section V.

II. Representation of a Noisy Two-Port Network

The wave representation of noise in a linear two-port has previously been described by the noiseless two-port and the two correlated noise sources shown in Fig. 1(a) [3]–[5]. We modify this approach by modeling the noise generated by the two-port as two correlated wave sources–one at each port—that radiate from the ports into matched loads. The complex emitted noise waves are designated as b_1 and b_2 in Fig. 1(b). The total noise wave at the output of the two-port is given by

$$b_{2_t} = b_1 \cdot \frac{\Gamma_s S_{21}}{1 - S_{11}\Gamma_s} + b_2 \tag{1}$$

where S_{11} and S_{21} are scattering coefficients of the two-port and Γ_s is the source reflection coefficient.

The average output noise power generated by the two-port is given by

$$\langle |b_{2_t}|^2 \rangle = A_{11}\left|\frac{\Gamma_s S_{21}}{1 - S_{11}\Gamma_s}\right|^2 + A_{22} + 2\,\mathrm{Re}\left\{A_{12}\frac{S_{21}\Gamma_s}{1 - S_{11}\Gamma_s}\right\} \tag{2}$$

where $A_{ij} = \langle b_i b_j^* \rangle$ are the elements of the correlation matrix of the noise sources. The diagonal elements A_{11} and A_{22} are real quantities representing the average power of each source, and the off-diagonal element A_{21} is complex and represents the correlation between sources. Following [5], we define noise temperatures for each element:

$$A_{11} = \langle |b_1|^2 \rangle = kT_1\,\Delta f$$
$$A_{22} = \langle |b_2|^2 \rangle = kT_2\,\Delta f \quad \text{and}$$
$$A_{12} = \langle b_1 b_2^* \rangle = kT_{12}\,\Delta f e^{j\phi_{12}}. \tag{3}$$

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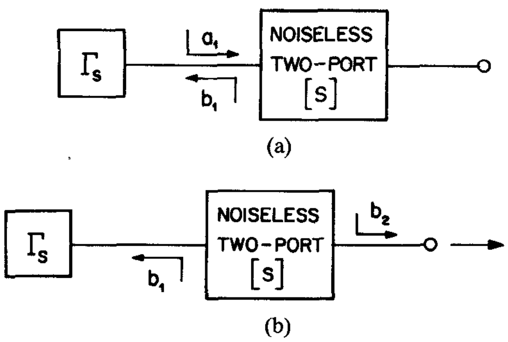

Fig. 1. The equivalent representation of a linear noisy two-port using the noise waves model. (a) Used by [3]–[5]. (b) Used in this paper.

In addition, a total noise temperature T_t can be defined as

$$\langle |b_{2_t}|^2 \rangle = kT_t \Delta f \tag{4}$$

where k is Boltzmann's constant and Δf is the noise bandwidth. Substituting (3) and (4) into (2) gives the total output noise temperature due only to noise generated inside the two-port as

$$T_t = T_1 \left| \frac{\Gamma_s S_{21}}{1 - S_{11}\Gamma_s} \right|^2 + T_2 + 2\,\mathrm{Re}\left\{ T_{12} e^{j\phi_{12}} \frac{\Gamma_s S_{21}}{1 - S_{11}\Gamma_s} \right\}. \tag{5}$$

Thus, T_t is a function of the source reflection coefficient Γ_s with four unknown real quantities: T_1, T_2, T_{12}, and ϕ_{12}. These unknowns can be obtained through measurements by varying the source reflection coefficient and observing T_t in a manner similar to that described by Meys [5] and Meys *et al.*, [7], [8]. For example, the measured value of T_t would be T_2 with a matched source ($|\Gamma_s| = 0$). For small values of $|\Gamma_s|$, T_t varies sinusoidally with the argument of Γ_s. The mean value of T_t is T_2, and the amplitude of the sinusoidal variation about T_2 is proportional to T_{12}. The quantity ϕ_{12} can be determined by measuring the phase of the sinusoidal variation.

The spot noise figure F can be determined once the elements of the noise correlation matrix are known. The noise figure is given by

$$(F-1)kT_0 \Delta f G_a = \frac{A_{11} \left| \frac{\Gamma_s S_{21}}{1 - S_{11}\Gamma_s} \right|^2 + A_{22} + 2\,\mathrm{Re}\left\{ A_{12} \frac{\Gamma_s S_{21}}{1 - S_{11}\Gamma_s} \right\}}{1 - |\Gamma_2|^2} \tag{6}$$

where Γ_2 is the reflection coefficient looking into the output port when the source is connected to the input. G_a is the available gain of the two-port, given by

$$G_a = \frac{(1 - |\Gamma_s|^2)|S_{21}|^2}{(1 - |\Gamma_2|^2)|1 - S_{11}\Gamma_s|^2} \tag{7}$$

and T_0 is the reference temperature.

III. Noise Analysis of Passive Multiports

Lossy passive networks are assumed to introduce only thermal noise. This makes the computation of the noise correlation matrix for a passive multiport easier than for

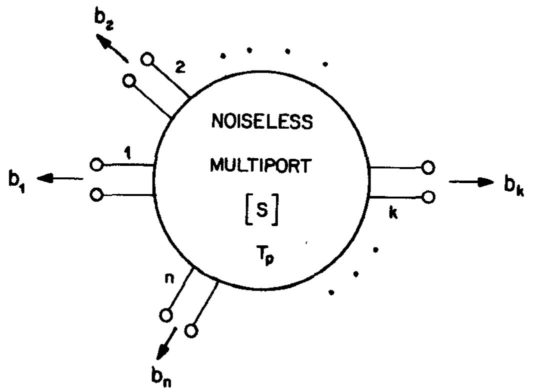

Fig. 2. The wave representation of noise in a passive multiport at uniform physical temperature.

an active two-port. The equivalent noise waves representing the thermal noise generated by a network having n ports are shown in Fig. 2 as $b_1, b_2, \cdots, b_n$. The $n \times n$ correlation matrix $\boldsymbol{P}$ of these noise sources is given by

$$\begin{aligned} \boldsymbol{P} &= [\langle b_i b_j^* \rangle], \qquad i, j = 1, 2, \cdots, n \\ &= k\,\Delta f T_p \boldsymbol{N} \end{aligned} \tag{8}$$

where T_p is the physical temperature of the multiport network and

$$\boldsymbol{N} = \boldsymbol{I} - \boldsymbol{S}\boldsymbol{S}^{\dagger}. \tag{9}$$

Here, $\boldsymbol{I}$ is the unit matrix, $\boldsymbol{S}$ is the scattering matrix of the n-port, and $\boldsymbol{S}^{\dagger}$ is the transposed complex conjugate of $\boldsymbol{S}$. Bosma [9] refers to $\boldsymbol{N}$ as the noise distribution matrix because it describes the distribution of the noise power generated within a passive multiport over its output ports.

We note that the physical temperature T_p of a multiport network may not be equal to the reference temperature T_0, as is often assumed. Furthermore, in a large system, the individual passive networks are not always at the same temperature. Consequently, our analysis allows each passive network to have its own physical temperature.

How can the physical temperature be obtained? We propose a possible method of calculating T_p from measurements.

If all ports of the n-port in Fig. 2, except i and j, are terminated in matched loads, G_{ij} is the available gain from port j to port i defined by (7). Assuming that the matched loads are in thermodynamic equilibrium with the multiport network, the temperature of each load is T_p under these conditions. The total output noise temperature T_{ij} available at port i is given by [10]

$$T_{ij} = T_p \left(1 - \sum_{\substack{m=1 \\ m \neq i}}^{n} G_{im} \right) + T_p \sum_{\substack{m=1 \\ m \neq i, j}}^{n} G_{im}. \tag{10}$$

The first term on the right-hand side of (10) represents the contribution from the network-generated noise while the second term is the contribution from the matched loads. Expansion of this expression leads to

$$T_{ij} = (1 - G_{ij}) T_p. \tag{11}$$

With the above arrangement, the noise figure from port j to port i can be expressed as

$$F_{ij} = 1 + \frac{kT_{ij}\Delta f}{kT_0 \Delta f G_{ij}}. \tag{12}$$

Combining (12) with (13), we obtain T_p as

$$T_p = \frac{(F_{ij}-1)T_0 G_{ij}}{1-G_{ij}} \tag{13}$$

and the correlation matrix $\boldsymbol{P}$ for a passive multiport can be specified completely.

IV. System of Arbitrary Topology

Once the noise correlation matrices of constituent multiport networks of a system are obtained, the overall correlation matrix of the resultant waves at the external ports can be calculated in terms of the individual correlation matrices and the scattering parameters. A general connection of arbitrary multiport component networks is shown in Fig. 3. Let the total number of component networks be m and the number of external ports in the overall system be n. In order for the network topology to be absolutely general, we assume that the component networks can have any number of ports.

In Fig. 3, $B_1, B_2, \cdots, B_n$ are the equivalent noise waves emitted from the external ports that represent the total noise generated within the overall system. The noise waves representing the noise generated by individual networks are designated as b_r^j, where the superscript j represents the network number and the subscript r represents the port of this network from which the wave emerges.

A resultant wave from the kth external port, B_k, consists of the contributions from all the components in the network, and is given by

$$B_k = \sum_{p=1}^{p_1} b_p^1 S_{kp'} + \sum_{p=1}^{p_2} b_p^2 S_{kp'} + \cdots + \sum_{p=1}^{p_m} b_p^m S_{kp'} \tag{14}$$

where $\sum_{p=1}^{p_j}$ represents the summation over all the p_j ports of the jth network. $S_{kp'}$ is the transmission coefficient from the port connected to port p of that network (designated in Fig. 3 as p') to the external port k. Note that $S_{kp'}$ includes all the possible transmission paths from port p' to port k, including reflection at port p'. In order to compute the resultant wave, B_k, a CAD program must be able to calculate the overall transmission coefficients from every internal port to the kth external port.

The equivalent noise wave sources of a network are, in general, correlated with each other but they are uncorrelated with other network noise sources. With this in mind, the cross correlation of the waves B_k and B_l at the external ports k and l can be obtained from (14) as

$$\langle B_k B_l^* \rangle = \sum_r^{p_1} \sum_p^{p_1} \langle b_p^1 b_r^{1*} \rangle S_{kp'} S_{lr'}^* + \cdots + \sum_r^{p_m} \sum_p^{p_m} \langle b_p^m b_r^{m*} \rangle S_{kp'} S_{lr'}^*. \tag{15}$$

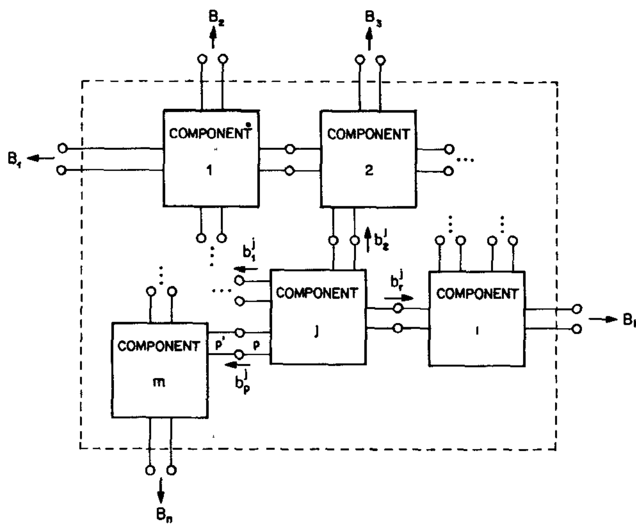

Fig. 3. A general connection of m linear noisy multiports.

To express (15) in a matrix form, let us introduce the following notation:

- $\boldsymbol{b}^j$ correlation matrix of noise waves for network j,
- $\boldsymbol{S}_k^j$ row matrix of transmission coefficients from all the ports connected to the ports of network j to the external port k.

With the above notation, (15) can be expressed as

$$\langle B_k B_l^* \rangle = \sum_{j=1}^{m} \boldsymbol{S}_k^j \cdot \boldsymbol{b}^j \cdot \boldsymbol{S}_l^{j\dagger} \tag{16}$$

and the noise correlation matrix $\boldsymbol{A}$ of the overall system is given by

$$\boldsymbol{A} = [\langle B_k B_l^* \rangle], \qquad k,l = 1,2,\cdots,n. \tag{17}$$

V. An Example

The algorithm described in the previous section was implemented as part of a general-purpose microwave system analysis program developed at the University of Massachusetts. The program is called CAAMS (computer-aided analysis of microwave systems). CAAMS, which runs on a VAX 11/750 computer, reduces the network topology by eliminating one interconnection (i.e., two ports) at a time in the manner described by Filipsson [11]. This procedure eliminates the need for matrix inversions and results in an efficient use of the CPU time. Also, the interconnections are eliminated in such a way that the resultant network has a minimum number of external ports at each stage of the computation. The intermediate values of the calculated scattering parameters are updated to yield the transmission coefficients $S_{kp'}$ in (14) for each interconnection.

As an example, we calculated the noise figure of the simple system shown in Fig. 4. The S-parameters for each component were measured from 8 to 16 GHz in steps of 1 GHz on an HP8510 automatic network analyzer and then read into the program through an interface between the network analyzer and the VAX computer. The noise

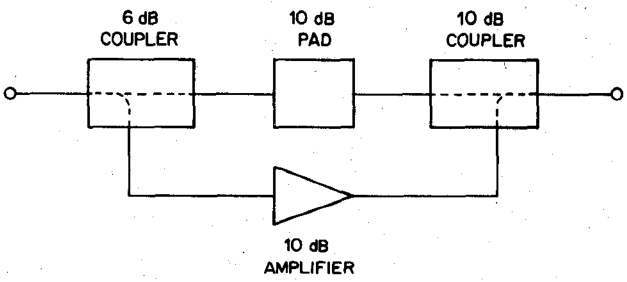

Fig. 4. Block diagram of the subsystem used as an example.

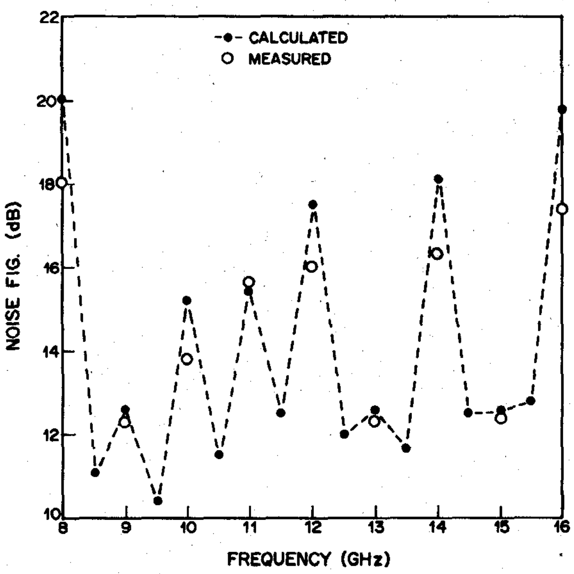

Fig. 5. Noise figure of the subsystem depicted in Fig. 4.

figures for each network were measured at the above frequencies, as was the overall system noise figure. The measured and calculated results are shown in Fig. 5. The networks and topology of our example were chosen so as to investigate the effects of interstage mismatches and transmission line lengths on the overall system noise figure. CAAMS is equipped with a routine to perform the sensitivity analysis on any network in the system. The user can vary the signal and noise parameters of a network in increments to observe the effects on overall system performance.

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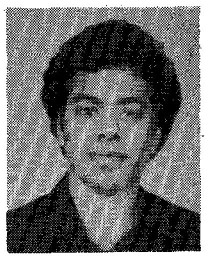

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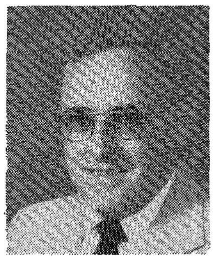

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